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Korovkin type approximation theorem for functions of two variables through statistical *A*-summability

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Abstract

In this article, we prove a Korovkin type approximation theorem for a function of two variables by using the notion of statistical *A*-summability. We also study the rate of statistical *A*-summability of positive linear operators. Finally we construct an example by Bleimann et al. operators to show that our result is stronger than those of previously proved by other authors.

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1 Introduction and preliminaries

The concept of statistical convergence for sequences of real numbers was introduced by Fast [1] and further studied many others.

Let $K \subseteq \mathbb{N}$ and $K_n = \{k \le n : k \in K\}$. Then the *natural density* of K is defined by $\delta(K) = \lim_n n^{-1} |K_n|$ if the limit exists, where $|K_n|$ denotes the cardinality of K_n.

A sequence $x = (x_k)$ of real numbers is said to be *statistically convergent* to L provided that for every $\varepsilon > 0$ the set $K_\varepsilon := \{k \in \mathbb{N} : |x_k - L| \ge \varepsilon\}$ has natural density zero, i.e. for each $\varepsilon > 0$,

$$\lim_n \frac{1}{n} |\{k \le n : |x_k - L| \ \ge \varepsilon\}| \ = 0.$$

In this case we write $st\text{-}\lim x = L$. Note that if $x = (x_k)$ is convergent then it is statistically convergent but not conversely. The idea of statistical convergence of double sequences has been intruduced and studied in [2,3].

Let $A = (a_{nk})$, $n, k \in \mathbb{N}$, be an infinite matrix and $x = (x_k)$ be a sequence. Then the (transformed) sequence, $Ax := (y_n)$, is denoted by

$$y_n := \sum_{k=1}^{\infty} a_{nk} x_k,$$

where it is assumed that the series on the right converges for each $n \in \mathbb{N}$. We say that a sequence x is A-summable to the limit ℓ if $y_n \to \ell$ as $n \to \infty$.

A matrix transformation is said to be regular if it maps every convergent sequence into a convergent sequence with the same limit. The well-known conditions for two dimensional matrix to be regular are known as Silverman-Toeplitz conditions.

In [4], Edely and Mursaleen have given the notion of statistical A-summability for single sequences and statistical A-summability for double sequences has recently been studied in [5].

Let $A = (a_{nk})$ be a nonnegative regular summability matrix and $x = (x_k)$ be a sequence of real or complex sequences. We say that x is statistically A-summable to L if for every $\varepsilon > 0$,

$$\delta\left(\left\{n \in \mathbb{N} : \left|y_n - L\right| \geq \varepsilon\right\}\right) = 0.$$

So, if x is statistically A-summable to L then for every $\varepsilon > 0$,

$$\lim_m \frac{1}{m}\left|\left\{n \leq m : \left|y_n - L\right| \geq \varepsilon\right\}\right| = 0.$$

Note that if a sequence is bounded and A-statistically convergent to L, then it is A-summable to L; hence it is statistically A-summable to L but not conversely (see [4]).

Example 1.1. Let $A = (C, 1)$, the Cesàro matrix and the sequence $u = (u_k)$ be defined by

$$u_k = \begin{cases} 1 & \text{if k is odd,} \\ 0 & \text{if k is even.} \end{cases} \tag{1.1}$$

Then u is is A-summable to $\frac{1}{2}$ (and hence statistically A-summable to 1/2) but not statistically (and not A-statistically as well) convergent.

Let $I := [0, \infty)$ and $C(I)$ denote the space of all continuous real valued functions on I. Let $C_B(I) := \{f \in C(I): f \text{ is bounded on } I\}$.$C(I)$ and $C_B(I)$ are equipped with norm

$$||f||_{C(I)} := \sup_{x \in I} |f(x)|.$$

Let $H_\omega(I)$ denote the space of all real valued functions f on I such that

$$|f(s) - f(x)| \leq \omega\left(f; \left|\frac{s}{1+s} - \frac{x}{1+x}\right|\right),$$

where ω is the modulus of continuity, i.e.

$$\omega(f; \delta) = \sup_{s,x \in I}\{|f(s) - f(x)| : |s - x| \leq \delta\}.$$

It is to be noted that any function $f \in H_\omega(I)$ is continuous and bounded on I.

The following Korovkin type theorem (see [6]) was proved by Çakar and Gadjiev [7].

Theorem A. Let (L_n) be a sequence of positive linear operators from $H_\omega(I)$ into $C_B(I)$. Then for all $f \in H_\omega(I)$

$$\lim_{n\to\infty} \left\|L_n(f; x) - f(x)\right\|_{C_B(I)} = 0$$

if and only if

$$\lim_{n\to\infty} \left\|L_n(f_i; x) - g_i\right\|_{C_B(I)} = 0 \ (i = 0, 1, 2),$$

where

$$g_0(x) = 1, g_1(x) = \frac{x}{1+x}, g_2(x) = \left(\frac{x}{1+x}\right)^2.$$

Erkuş and Duman [8] have given the st_A-version of the above theorem for functions of two variables.Quite recently, Korovkin type of approximation theorems have been proved in [9,10] by using almost convergence; in [11-15] by using variants of statistical convergence and in [16-19] for functions of two variables by using statistical convergence, A-statistical convergence and statistical A-summability of double sequences. In this article, we use the notion of statistical A-summability to prove a Korovkin type approximation theorem for functions of two variables with the help of test functions $1, \frac{x}{1+x}, \frac{y}{1+y}, \left(\frac{x}{1+x}\right)^2 + \left(\frac{y}{1+y}\right)^2$.

2 Main result

Let $I = [0, \infty)$ and $K = I \times I$. We denote by $C_B(K)$ the space of all bounded and continuous real valued functions on K equipped with norm

$$||f||_{C_B(K)} := \sup_{(x,y)\in K} |f(x,y)|, \quad f \in C_B(K).$$

Let $H_{\omega^*}(K)$ denote the space of all real valued functions f on K such that

$$|f(s,t) - f(x,y)| \leq \omega^* \left(f; \sqrt{\left(\frac{s}{1+s} - \frac{x}{1+x}\right)^2 + \left(\frac{t}{1+t} - \frac{y}{1+y}\right)^2} \right),$$

where ω^* is the modulus of continuity, i.e.

$$\omega^*(f;\delta) = \sup_{(s,t),(x,y)\in K} \left\{ |f(s,t) - f(x,y)| \ : \sqrt{(s-x)^2 + (t-y)^2} \leq \delta \right\}.$$

It is to be noted that any function $f \in H_{\omega^*}(K)$ is bounded and continuous on K, and a necessary and sufficient condition for $f \in H_{\omega^*}(K)$ is that

$$\lim_{\delta\to 0} \omega^*(f;\delta) = 0.$$

We prove the following result:

Theorem 2.1. Let $A = (a_{nk})$ be nonnegative regular summability matrix. Let (T_k) be a sequence of positive linear operators from $H_{\omega^*}(K)$ into $C_B(K)$. Then for all $f \in H_{\omega^*}(K)$

$$st\text{ - }\lim_{n\to\infty} \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f;x,y) - f(x,y) \right\|_{C_B(K)} = 0 \tag{2.0}$$

if and only if

$$st\text{ - }\lim_{n\to\infty} \left\| \sum_{k=1}^{\infty} a_{nk} T_k(1;x,y) - 1 \right\|_{C_B(K)} = 0 \tag{2.1}$$

$$st\text{ - }\lim_{n\to\infty} \left\| \sum_{k=1}^{\infty} a_{nk} T_k\left(\frac{s}{1+s};x,y\right) - \frac{x}{1+x} \right\|_{C_B(K)} = 0 \tag{2.2}$$

$$st\text{ - }\lim_{n\to\infty} \left\| \sum_{k=1}^{\infty} a_{nk} T_k\left(\frac{t}{1+t};x,y\right) - \frac{y}{1+y} \right\|_{C_B(K)} = 0 \tag{2.3}$$

$$st\text{ - }\lim_{n\to\infty}\left\|\sum_{k=1}^{\infty}a_{nk}T_k\left(\left(\frac{s}{1+s}\right)^2+\left(\frac{t}{1+t}\right)^2;x,y\right)-\left(\left(\frac{x}{1+x}\right)^2+\left(\frac{y}{1+y}\right)^2\right)\right\|_{C_B(K)}=0. \quad (2.4)$$

Proof. Since each of the functions $f_0(x,y)=1, f_1(x,y)=\frac{x}{1+x}, f_2(x,y)=\frac{y}{1+y}, f_3(x,y)=\left(\frac{x}{1+x}\right)^2+\left(\frac{y}{1+y}\right)^2$ belongs to $H_{\omega^*}(K)$, conditions (2.1)-(2.4) follow immediately from (2.0).

Let $f\in H_{\omega^*}(K)$ and $(x, y)\in K$ be fixed. Then for $\varepsilon>0$ there exist δ_1, $\delta_2>0$ such that $|f(s,t)-f(x,y)|<\varepsilon$ holds for all $(s,t)\in K$ satisfying $\left|\frac{s}{1+s}-\frac{x}{1+x}\right|<\delta_1$ and $\left|\frac{t}{1+t}-\frac{y}{1+y}\right|<\delta_2$.

Let

$$K(\delta):=\left\{(s,t)\in K:\sqrt{\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2}<\delta=\min\{\delta_1,\delta_2\}\right\}.$$

Hence

$$\begin{aligned}|f(s,t)-f(x,y)|=&\ |f(s,t)-f(x,y)|\chi_{K(\delta)}(s,t)+|f(s,t)-f(x,y)|\chi_{K\setminus K(\delta)}(s,t)\\ &\le\varepsilon+2N\chi_{K\setminus K(\delta)}(s,t)\end{aligned} \quad (2.5)$$

Where χ_D denotes the characteristic function of the set D and $N=||f||_{C_B(K)}$. Further we get

$$\chi_{K\setminus K(\delta)}(s,t)\le\frac{1}{\delta_1^2}\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\frac{1}{\delta_2^2}\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2. \quad (2.6)$$

Combining (2.5) and (2.6), we get

$$|f(s,t)-f(x,y)|\ \le\varepsilon+\frac{2N}{\delta^2}\left\{\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2\right\}, \quad (2.7)$$

After using the properties of f, a simple calculation gives that

$$\begin{aligned}|T_k(f;x,y)-f(x,y)|\ &\le\varepsilon+M\{|T_k(f_0;x,y)-f_0(x,y)|+|T_k(f_1;x,y)-f_1(x,y)|\\ &+|T_k(f_2;x,y)-f_2(x,y)|+|T_k(f_3;x,y)-f_3(x,y)|\},\end{aligned} \quad (2.8)$$

where

$$M:=\varepsilon+N+\frac{4N}{\delta^2}.$$

Now replacing $T_k(f;x,y)$ by $\sum_{k=1}^{\infty}a_{nk}T_k(f;x,y)$ and taking $\sup_{(x,y)\in K}$, we get

$$\begin{aligned}\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f;x,y)-f(x,y)\right\|_{C_B(K)}&\le\varepsilon+M\left(\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right\|_{C_B(K)}\right.\\ &+\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f_1;x,y)-f_1(x,y)\right\|_{C_B(K)}+\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f_2;x,y)-f_2(x,y)\right\|_{C_B(K)}\\ &\left.+\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f_3;x,y)-f_3(x,y)\right\|_{C_B(K)}\right).\end{aligned} \quad (2.9)$$

For a given $r > 0$ choose $\varepsilon > 0$ such that $\varepsilon < r$. Define the following sets

$$D := \left\{ n : \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f; x, y) - f(x, y) \right\|_{C_B(K)} \geq r \right\},$$

$$D_1 := \left\{ n : \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f_0; x, y) - f_0(x, y) \right\|_{C_B(K)} \geq \frac{r - \varepsilon}{4K} \right\},$$

$$D_2 := \left\{ n : \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f_1; x, y) - f_1(x, y) \right\|_{C_B(K)} \geq \frac{r - \varepsilon}{4K} \right\},$$

$$D_3 := \left\{ n : \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f_2; x, y) - f_2(x, y) \right\|_{C_B(K)} \geq \frac{r - \varepsilon}{4K} \right\},$$

$$D_4 := \left\{ n : \left\| \sum_{k=1}^{\infty} a_{nk} T_k(f_3; x, y) - f_3(x, y) \right\|_{C_B(K)} \geq \frac{r - \varepsilon}{4K} \right\}.$$

Then from (2.9), we see that $D \subset D_1 \cup D_2 \cup D_3 \cup D_4$ and therefore $\delta(D) \leq \delta(D_1) + \delta(D_2) + \delta(D_3) + \delta(D_4)$. Hence conditions (2.1)-(2.4) imply the condition (2.0).

This completes the proof of the theorem.

If we replace the matrix A in Theorem 2.1 by identity matrix, then we immediately get the following result which is due to Erkuş and Duman [8]:

Corollay 2.2. Let $A = (a_{nk})$ be nonnegative regular summability matrix. Let (T_k) be a sequence of positive linear operators from $H_{\omega^*}(K)$ into $C_B(K)$. Then for all $f \in H_{\omega^*}(K)$

$$st\text{-}\lim_{k\to\infty} \left\| T_k(f; x, y) - f(x, y) \right\|_{C_B(K)} = 0 \tag{2.10}$$

if and only if

$$st\text{-}\lim_{k\to\infty} \left\| T_k(1; x, y) - 1 \right\|_{C_B(K)} = 0, \tag{2.11}$$

$$st\text{-}\lim_{k\to\infty} \left\| T_k\left(\frac{s}{1+s}; x, y\right) - \frac{x}{1+x} \right\|_{C_B(K)} = 0, \tag{2.12}$$

$$st\text{-}\lim_{k\to\infty} \left\| T_k\left(\frac{t}{1+t}; x, y\right) - \frac{y}{1+y} \right\|_{C_B(K)} = 0, \tag{2.13}$$

$$st\text{-}\lim_{k\to\infty} \left\| T_k\left(\left(\frac{s}{1+s}\right)^2 + \left(\frac{t}{1+t}\right)^2; x, y\right) - \left(\left(\frac{x}{1+x}\right)^2 + \left(\frac{y}{1+y}\right)^2\right) \right\|_{C_B(K)} = 0. \tag{2.14}$$

3 Statistical rate of convergence

In this section, using the concept of statistical A-summability we study the rate of convergence of positive linear operators with the help of the modulus of continuity. Let us recall, for $f \in H_{\omega^*}(K)$

$$|f(s, t) - f(x, y)| \leq \omega^*\left(f; \sqrt{\left(\frac{s}{1+s} - \frac{x}{1+x}\right)^2 + \left(\frac{t}{1+t} - \frac{y}{1+y}\right)^2}\right),$$

where

$$\omega^*(f;\delta)=\sup_{(s,t),(x,y)\in K}\left\{|f(s,t)-f(x,y)|\ :\sqrt{(s-x)^2+(t-y)^2}\le\delta\right\}.$$

We have the following result:

Theorem 3.1. Let $A=(a_{nk})$ be nonnegative regular summability matrix. Let (T_k) be a sequence of positive linear operators from $H_{\omega^*}(K)$ into $C_B(K)$. Assume that

(i) st - $\lim_{n\to\infty}\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f_0)-f_0\right\|_{C_B(K)}=0,$

(ii) st- $\lim_{n\to 0}\omega^*(f;\delta_n)=0$, where

$$\delta_n=\sqrt{\left\|\sum_{k=1}^{\infty}a_{nk}T_k(\psi)\right\|_{C_B(K)}}\text{ with }\psi=\psi(s,t)=\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2.$$

Then for all $f\in H_{\omega^*}(K)$

$$st\text{ - }\lim_{n\to\infty}\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f)-f\right\|_{C_B(K)}=0.$$

Proof. Let $f\in H_{\omega^*}(K)$ be fixed and $(x,y)\in K$ be fixed. Using linearity and positivity of the operators T_k for all $n\in\mathbb{N}$, we have

$$\begin{aligned}
\left|\sum_{k=1}^{\infty}a_{nk}T_k(f;x,y)-f(x,y)\right| &\le \sum_{k=1}^{\infty}a_{nk}T_k(|f(s,t)-f(x,y)|;x,y)\\
&\quad+|f(x,y)|\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right|\\
&\le\sum_{k=1}^{\infty}a_{nk}T_k\left(\omega^*\left(f;\delta\frac{\sqrt{\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2}}{\delta}\right);x,y\right)\\
&\quad+||f||_{C_B(K)}\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right|\\
&\le\sum_{k=1}^{\infty}a_{nk}T_k\left(\left(1+\left[\frac{\sqrt{\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2}}{\delta}\right]\right)\omega^*(f;\delta);x,y\right)\\
&\quad+||f||_{C_B(K)}\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right|\\
&\le\sum_{k=1}^{\infty}a_{nk}\omega^*(f;\delta)T_k\left(\left(1+\frac{\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2}{\delta^2}\right);x,y\right)\\
&\quad+||f||_{C_B(K)}\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right|\\
&\le\omega^*(f;\delta)\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\ \right|+||f||_{C_B(K)}\left|\sum_{k=1}^{\infty}a_{nk}T_k(f_0;x,y)-f_0(x,y)\right|\\
&\quad+\omega^*(f;\delta)+\frac{\omega^*(f;\delta)}{\delta^2}\sum_{k=1}^{\infty}a_{nk}T_k\left(\left(\left(\frac{s}{1+s}-\frac{x}{1+x}\right)^2+\left(\frac{t}{1+t}-\frac{y}{1+y}\right)^2;x,y\right)\right).
\end{aligned}$$

Hence

$$\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f) - f\right\|_{C_B(K)}.$$

$$\leq \ ||f||_{C_B(K)} \left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)} + \omega^*(f;\delta)\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)}$$

$$+ \frac{\omega^*(f;\delta)}{\delta^2}\left\|\sum_{k=1}^{\infty} a_{nk}T_k(\psi)\right\|_{C_B(K)} + \omega^*(f;\delta).$$

Now if we choose $\delta := \delta_n := \sqrt{\left\|\sum_{k=1}^{\infty} a_{nk}T_k(\psi)\right\|_{C_B(K)}}$ then

$$\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f) - f\right\|_{C_B(K)}.$$

$$\leq \ ||f||_{C_B(K)} \left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)}$$

$$+ \omega^*(f;\delta_n)\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)} + 2\omega^*(f;\delta_n).$$

Therefore

$$\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f) - f\right\|_{C_B(K)}.$$

$$\leq M\left\{\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)}\right. \tag{3.1}$$

$$\left. +\omega^*(f;\delta_n)\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0) - f_0\right\|_{C_B(K)} + \omega^*(f;\delta_n)\right\},$$

where $M = \max\{2, ||f||_{C_B(K)}\}$. Now, for a given $r > 0$, choose $\varepsilon > 0$ such that $\varepsilon > r$. Let us write

$$E := \left\{n : \left\|\sum_{k=1}^{\infty} a_{nk}T_k(f;x,y) - f(x,y)\right\|_{C_B(K)} \geq r\right\},$$

$$E_1 := \left\{n : \left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0;x,y) - f_0(x,y)\right\|_{C_B(K)} \geq \frac{r}{3K}\right\},$$

$$E_2 := \left\{n : \omega^*(f;\delta_n) \geq \frac{r}{3K}\right\},$$

$$E_3 := \left\{n : \omega^*(f;\delta_n)\left\|\sum_{k=1}^{\infty} a_{nk}T_k(f_0;x,y) - f_0(x,y)\right\|_{C_B(K)} \geq \frac{r}{3K}\right\}.$$

Then $E \subset E_1 \cup E_2 \cup E_3$ and therefore $\delta(E) \leq \delta(E_1) + \delta(E_2) + \delta(E_3)$. Using conditions (i) and (ii) we conclude

$$st\text{ - }\lim_{n\to\infty}\left\|\sum_{k=1}^{\infty}a_{nk}T_k(f)-f\right\|_{C_B(K)}=0.$$

This completes the proof of the theorem.

4 Example and the concluding remark

We show that the following double sequence of positive linear operators satisfies the conditions of Theorem 2.1 but does not satisfy the conditions of Corollary 2.2 and Theorem 2.1 of [8].

Example 4.1. Consider the following Bleimann et al. [20] (of two variables) operators:

$$B_n(f;x,y):=\frac{1}{(1+x)^n(1+y)^n}\sum_{j=0}^{n}\sum_{k=0}^{n}f\left(\frac{j}{n-j+1},\frac{k}{n-k+1}\right)\binom{n}{j}\binom{n}{k}x^jy^k,\quad(4.1)$$

where $f\in H_\omega(K)$, $K=[0,\infty)\times[0,\infty)$ and $n\in\mathbb{N}$.

Since

$$(1+x)^n=\sum_{j=0}^{n}\binom{m}{j}x^j,$$

it is easy to see that

$$B_n(f_0;x,y)\to 1=f_0(x,y).$$

Also by simple calculation, we obtain

$$B_n(f_1;x,y)=\frac{n}{n+1}\left(\frac{x}{1+x}\right)\to\frac{x}{1+x}=f_1(x,y),$$

and

$$B_n(f_2;x,y)=\frac{n}{n+1}\left(\frac{y}{1+y}\right)\to\frac{y}{1+y}=f_2(x,y).$$

Finally, we get

$$\begin{aligned}&B_n(f_3;x,y)\\ &=\frac{n(n-1)}{(n+1)^2}\left(\frac{x}{1+x}\right)^2+\frac{n}{(n+1)^2}\left(\frac{x}{1+x}\right)\\ &\quad+\frac{n(n-1)}{(n+1)^2}\left(\frac{y}{1+y}\right)^2+\frac{n}{(n+1)^2}\left(\frac{y}{1+y}\right)\\ &\to\left(\frac{x}{1+x}\right)^2+\left(\frac{y}{1+y}\right)^2=f_3(x,y).\end{aligned}$$

Now, take $A=C(1,1)$ and define $u=(u_n)$ by (1.1). Let the operator $L_n:H_\omega(K)\to C_B(K)$ be defined by

$$L_n(f;x,y)=(1+u_n)B_n(f;x,y).$$

It is easy to see that the sequence (L_n) satisfies the conditions (2.1)-(2.4). Hence by Theorem 2.1, we have

$$st\text{ - }\lim_{m\to\infty}\left\|\sum_{n=1}^{\infty}a_{mn}L_n(f;x,y)-f(x,y)\right\|_{C_B(K)}$$
$$=st\text{ - }\lim_{m\to\infty}\left\|\frac{1}{m}\sum_{n=1}^{m}L_n(f;x,y)-f(x,y)\right\|_{C_B(K)}=0.$$

On the other hand, the sequence (L_n) does not satisfy the conditions of Theorem A and Corollary 2.2 and Theorem 2.1 of [8], since (L_n) is neither convergent nor statistically (nor A-statistically) convergent. That is, Theorem A, Corollary 2.2 and Theorem 2.1 of [8] do not work for our operators L_n. Hence our Theorem 2.1 is stronger than Corollary 2.2 and Theorem 2.1 of [8].

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Authors' contributions
The main result of the paper was proposed and proved by AA, Section 3 was given by MM, and Section 4 was designed by both the authors jointly. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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